Finite Element Method - Exercise 4

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Abstract

A FEM implementation of a rectangular grid, subdivided into triangles, was implemented in Julia. The general task was to solve the steady state heat conduction problem on a 2D domain by solving the differential equation with the constitutive law. A finite element model with Dirichlet and Neumann boundary conditions was implemented. The model was solved by different variations like stretching to a trapezoidal shape, biased mesh, transforming the shape to an annulus and modification of a section within the domain with two different materials.

1 Introduction

The mathematical formulation of the problem is based on the book [ZTZ13], especially Chapter 5 was used. Tasks and descriptions from [PS22] were used. The goal of this exercise was to implement a FEM code that solves the steady state heat conduction problem. The programming language used is Julia. The material (Silver, solid, face centred cubic) observed has a thermal conductivity k = 429 $\frac{W}{mK}$. At the top of the domain, there is a Dirichlet boundary condition with a fixed temperature of 293 Kelvin. At the bottom one has Neumann boundary condition with a flux of 2.000.000 $\frac{W}{m^2}$. At the left and right side of the domain the flux is zero. The domain size is 0.01 m in x- and y-direction, whereas the thickness of the material is 0.005 m. In the plots, the gradient is red, and flux is blue.

2 Solution of the standard FEM model

The first FEM solution shows the basic 2D domain with no changes. The heat flow moves from the top (heat source) to the bottom of the domain with a constant gradient/flux.

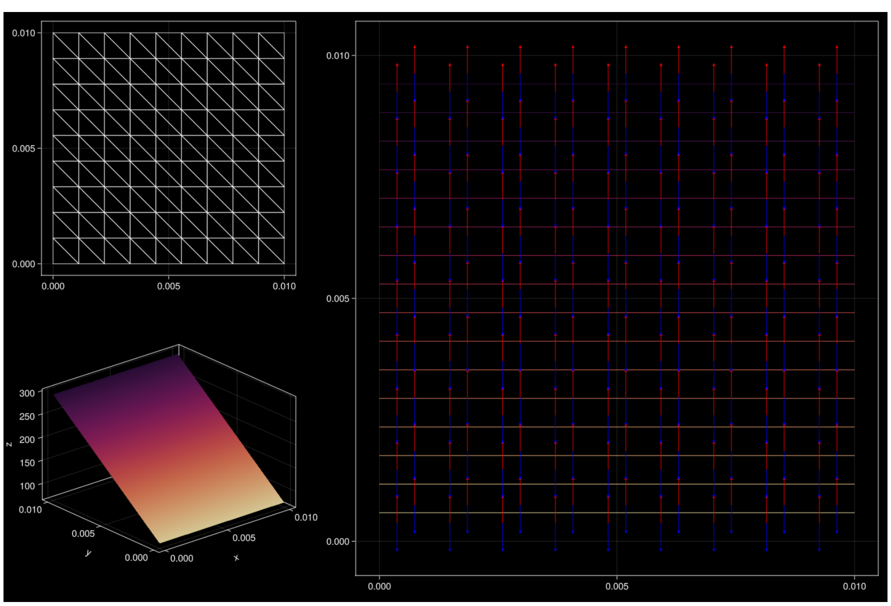

Figure 1: Standard 2D model.

3 Variations of the FEM model

By variation of the problem different shapes and defects were analysed.

3.1 Trapezoidal shape

By reshaping the domain, one compresses the triangles to get a trapezoidal shape. This was achieved by shifting the nodes towards the left based on their y-coordinate within the grid. With this new topology, the "tip" of the trapezoid in the lower right was the coldest part, since it is furthest away from the heat source. Following from the same logic, the bottom edge gets warmer towards the right.

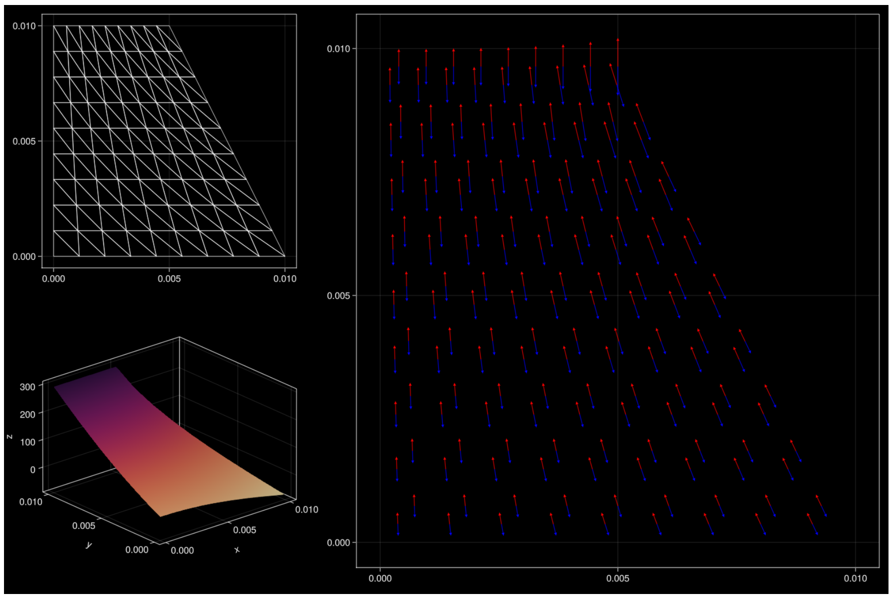

Figure 2: Reshape the domain by a trapezoidal shape.

3.2 Biased mesh

Biasing the mesh had seemingly no effect on the solution, which is what we would expect. Unfortunately, our plotting tools could not generate contour plots for irregular meshes.

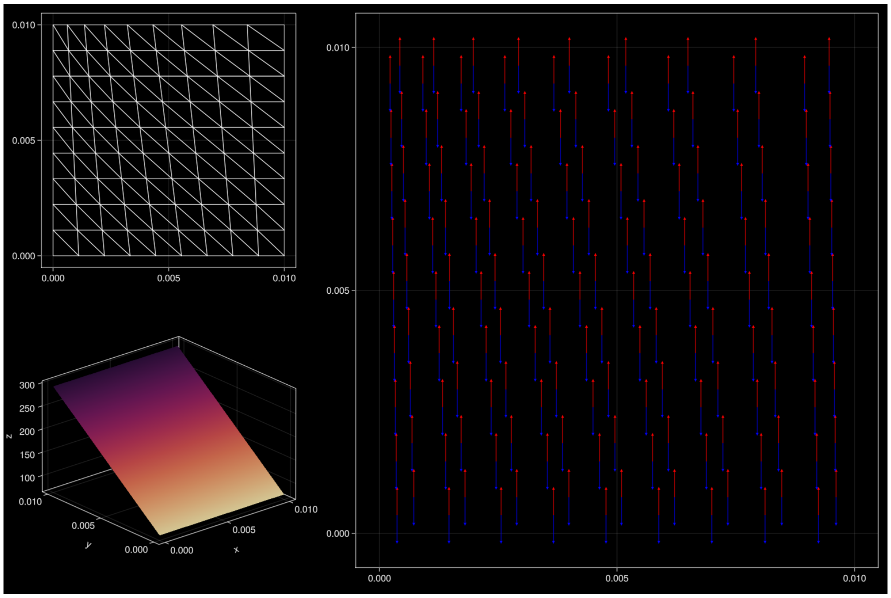

Figure 3: Biased mesh in x-direction.

3.3 Annulus shape

Transforming the rectangle into an annulus shape leads to results similar to the trapezoid. Along the lower edge, the side closer to the heat source has the highest temperature and the heat falls off towards the right.

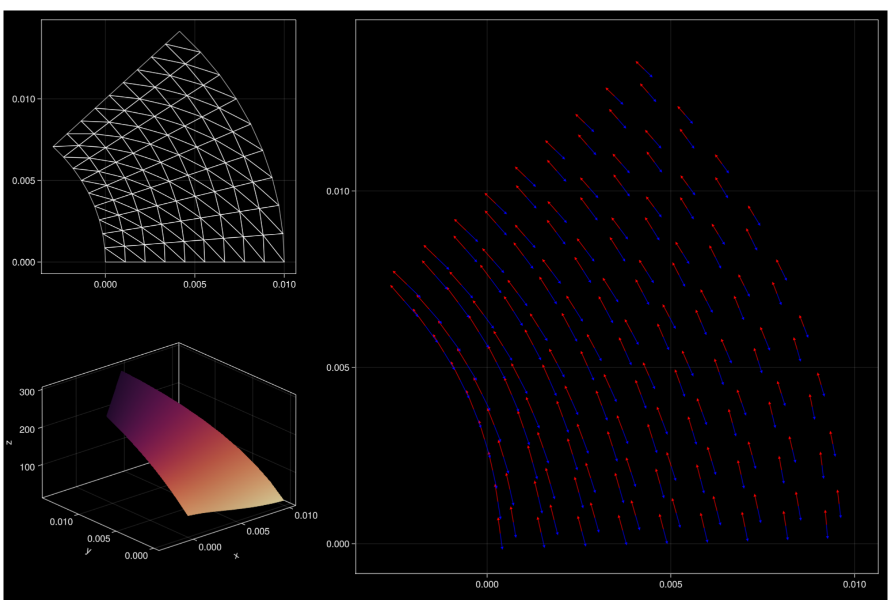

Figure 4: Reshape the domain by using annulus.

3.4 Subdomain with different heat conductivity

Within the domain a triangle of a different material was added to change the properties of the plane.

3.4.1 Increasing the heat conductivity of a triangular defect

When increasing the heat conductivity by a factor of c = 10 one gets a material with a thermal conductivity of 4290 - which corresponds to some sort of graphite (solid, honeycomb lattice). This leads to a faster heat transfer from top to bottom within the defect.

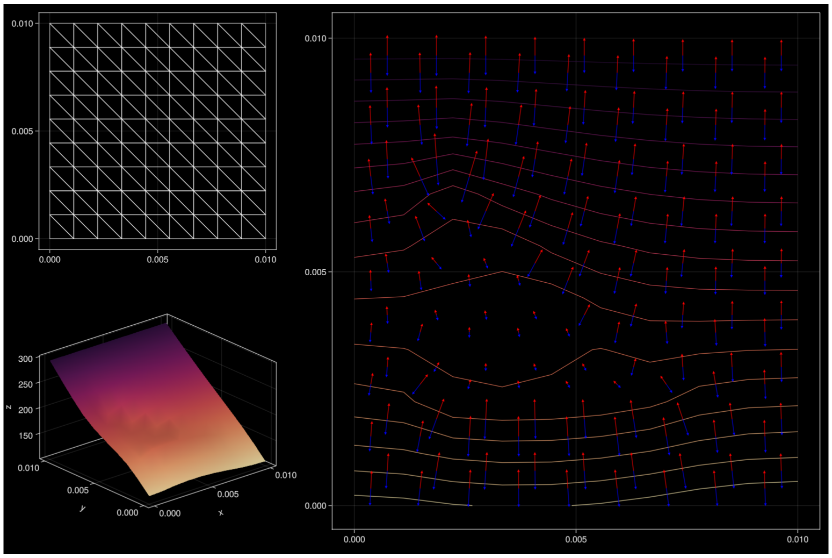

Figure 5: Triangular defect with higher heat conductivity than surrounding material.

3.4.2 Decreasing the heat conductivity of a triangular defect

When decreasing the heat conductivity by a factor of c = 10 one gets a material with a thermal conductivity of 42.9 - which corresponds to Rhenium (solid, hexagonal). This defect acts as an insulator and prevents the heat from moving faster through the domain at the position of the defect.

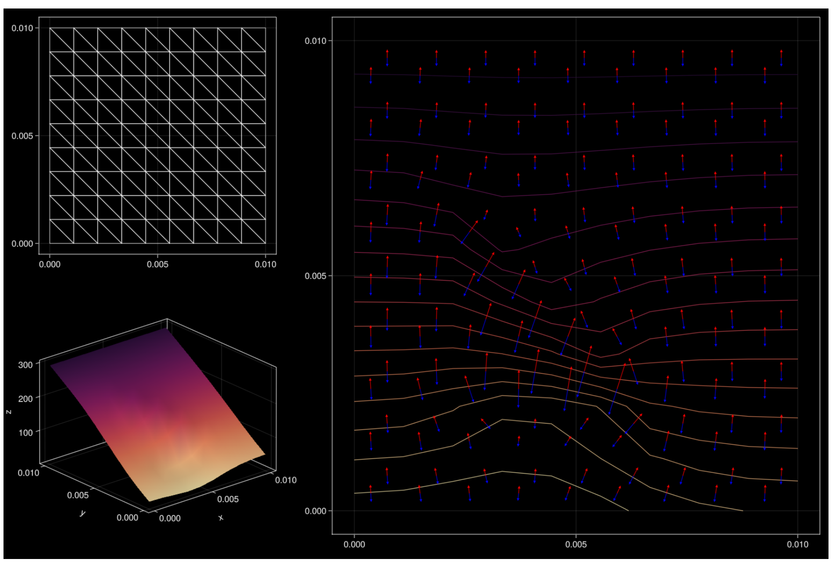

Figure 6: Triangular defect with lower heat conductivity than surrounding material.

References

[PS22] H. Pettermann and M. Schasching. Exercise 4 finite element method. *TU Vienna*, 2022.

[ZTZ13] O. Zienkiewicz, R. Taylor, and J. Zhu. *Finite Element Method: its Basis and Fundamentals*. Oxford: Butterworth-Heinemann, 7 edition, 2013.